Exercise 57

(a) Prove that the equation has at least one real root. (b) Use your calculator to find an interval of length 0.01 that contains a root.

$$\cos x = x^3$$

Solution

Bring both terms to the same side.

$$\cos x - x^3 = 0$$

The function $f(x) = \cos x - x^3$ is continuous everywhere because it's the difference of two functions known to be continuous everywhere, a trigonometric function and a polynomial function.

$$f(x) = 0$$

Find a value of x for which the function is negative, and find a value of x for which the function is positive.

$$f(0) = 1$$

$$f(1) \approx -0.460$$

$f(x)$ is continuous on the closed interval $[0, 1]$, and $N = 0$ lies between $f(0)$ and $f(1)$. By the Intermediate Value Theorem, then, there exists a root within $0 < x < 1$. Find other values of x within this interval for which the function is negative and positive.

$$f(0.8) \approx 0.185$$

$$f(0.9) \approx -0.107$$

$f(x)$ is continuous on the closed interval $[0.8, 0.9]$, and $N = 0$ lies between $f(0.8)$ and $f(0.9)$. By the Intermediate Value Theorem, then, there exists a root within $0.8 < x < 0.9$. Find other values of x within this interval for which the function is negative and positive.

$$f(0.86) \approx 0.0164$$

$$f(0.87) \approx -0.0137$$

$f(x)$ is continuous on the closed interval $[0.86, 0.87]$, and $N = 0$ lies between $f(0.86)$ and $f(0.87)$. By the Intermediate Value Theorem, then, there exists a root within $0.86 < x < 0.87$.